TECHNICAL NOTE

John N. Lomonte,[1] *M.S.*

Determination of Volumes in Laboratory Vessels

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ABSTRACT: Equations are derived to calculate volumes in partially filled round bottom and Erlenmeyer flasks by use of simple hand-held calculators. The calculations are made from dimensional measurements of the vessels and the height of the liquid inside the vessels. By making these measurements at the scene of clandestine laboratory seizures and taking representative samples, the excess quantities of materials seized may be taken directly to waste disposal without causing a hazardous materials storage problem

KEYWORDS: forensic science, criminalistics measurements, laboratory vessels

It has been a continuing problem for law enforcement and laboratory personnel to satisfy both the criminal code and the health and safety code when dealing with materials seized at clandestine laboratories. There needs to be a method to acquire sufficient evidence to sustain prosecution while allowing the disposal of the large amounts of hazardous materials encountered at these laboratories. As most law enforcement property rooms are poorly equipped for the storage of these materials, and are not allowed to do so, disposal of excess quantities directly from the laboratory scene is preferred. Under Texas law [*1*], it is permissible to destroy these excess quantities provided photographs are taken to show the amounts of these materials and to take representative samples for analysis. The total amount may be determined either by making actual volume measurements or calculation from dimensional measurements of the vessel and its contents.

Previous papers concerning volumes in laboratory vessels [*2,3*] used computers to handle the tedious calculations arising from the analytical geometry technique used and were limited to round bottom reaction vessels. This paper presents equations that allow the use of simple hand-held calculators to determine volumes from measurements made at the laboratory scene and extends the method to include Erlenmeyer flasks.

Mathematical Considerations

The round bottom flask and the Erlenmeyer flask can be considered mathematically to be solids of revolution of the appropriate geometrical shapes about an axis producing circular cross-sections. This cross-section can then be integrated with respect to the

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[1]Senior Chemist, Southwestern Institute of Forensic Sciences, Dallas, TX.

distance along that axis thus determining the volume of the solid for any value along the axis. This integral calculus needs only to be done once for a particular shape as it derives a simple general equation for calculating any partial volume.

If the geometric shape is revolved about the x-axis, then for any value of x, there will be a value for y, which is the radius of the circular cross-section and the area of that cross-section is πy^2. The volume of the solid of revolution is given by:

$$V = \int_{x=0}^{x=\max} \pi y^2 dx$$

The Round Bottom Flask

The round bottom flask can be considered as the sphere generated by revolving about the x-axis a circle tangent to the y-axis at the origin (Fig. 1). The equation of a circle whose center is not located at the origin is:

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h,k) are the coordinates of the center of the circle and r is the radius. In this case $h = r$ and $k = 0$, and the equation of the circle is:

$$(x - r)^2 + y^2 = r^2$$

At every value for x between $x = 0$ (flask empty) and $x = 2r$ (twice the radius—flask full), there is a corresponding value for y, which is the radius of the circular cross-section of the sphere at that value of x. Solving the equation of the circle for y^2 and simplifying:

$$y^2 = 2rx - x^2$$

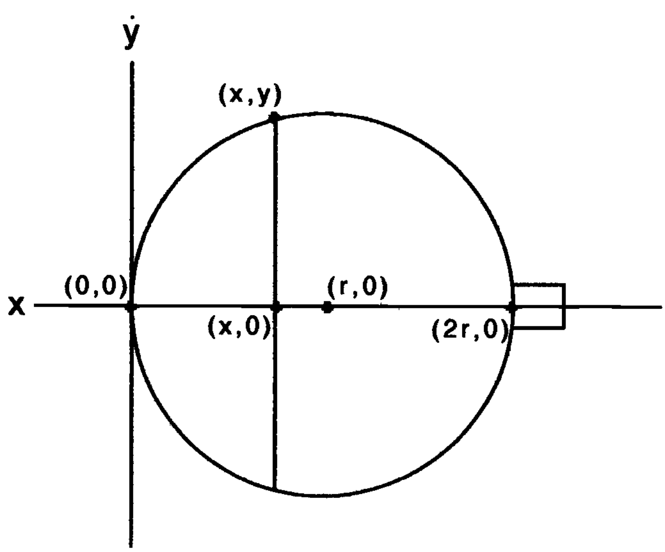

FIG. 1—*The round-bottom flask.*

Substitute into the general equation for the volume of a solid of revolution:

$$V = \int_{x=0}^{x=2r} \pi(2rx - x^2)dx$$

Integrate:

$$V = \pi\left(2\frac{rx^2}{2} - \frac{x^3}{3} + C\right)$$

At $x = 0$; $V = 0$; so $C = 0$.

Therefore: The volume of a partially filled round bottom flask can be found by:

$$V = \pi\left(rx^2 - \frac{x^3}{3}\right)$$

where

r = radius of flask in cm,
x = height of liquid in cm, and
V = volume of liquid in mL.

$$\textit{Proof: At } x = 2r; V = \frac{4}{3}\pi r^3$$

which is the formula for the volume of a sphere.

In practice, one simply places a measuring device such as a transfer pipet inside the flask and notes the level of the liquid and the height at which the neck of the flask intersects the body, thus measuring the diameter. The distances on the pipet are measured to the nearest millimetre. For double layers, calculate total liquid volume. Then calculate the volume of the lower layer from its height. The difference between the two values is the volume of the upper layer.

Example: A round bottom flask has a measured diameter of 17.9 cm and a liquid height of 12.3 cm. What is the volume of the liquid?

As diameter equals 17.9 cm, the radius (r) = 8.95 cm and x = liquid height = 12.3 cm.

$$V = \pi\left[8.95(12.3)^2 - \frac{(12.3)^3}{3}\right]$$

$$V = 2300\ mL$$

The Erlenmeyer Flask

The Erlenmeyer flask can be considered as the cone generated by revolving about the x-axis a straight line that intersects both axes (Fig. 2).

The equation of a line is:

$$y = mx + b$$

where m is the slope of the line and b is the y intercept. As the coordinates of the points at the ends of the line are $(0,r)$ for point 1 where r is the radius of the base of the flask

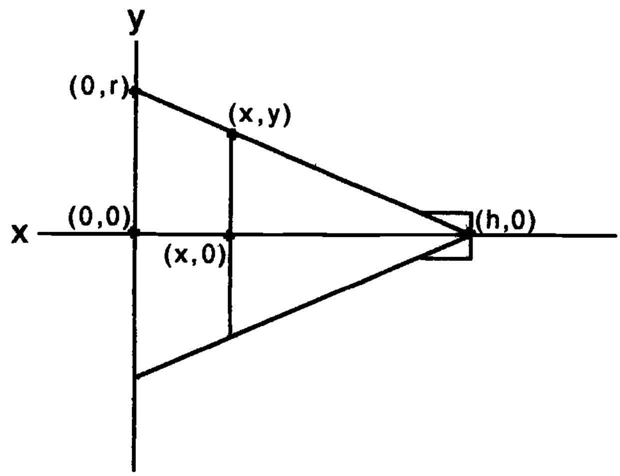

FIG. 2—*The Erlenmeyer flask.*

and $(h,0)$ for point 2 where h is the height of the flask, the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1};$$

or

$$m = \frac{0 - r}{h - 0} = -\frac{r}{h}.$$

The y intercept $b = r$, and the equation of the line is:

$$y = -\frac{rx}{h} + r.$$

If the line is revolved about the x-axis, a cone is generated. At every value for x between $x = 0$ (flask empty) and $x = h$ (height of vessel—flask full), there is a corresponding value for y, which is the radius of the circular cross-section of the cone at that value of x.

The equation of the line is squared so that y^2 can be determined for use in the general equation for the volume of a solid of revolution. After squaring, substitute into the integral:

$$V = \int_{x=0}^{x=h} \pi\left(\frac{r^2x^2}{h^2} - \frac{2r^2x}{h} + r^2\right)dx$$

Integrate:

$$V = \pi\left(\frac{r^2x^3}{3h^2} - \frac{2r^2x^2}{2h} + r^2x + C\right)$$

At $X = 0$; $V = 0$; so $C = 0$.

Factor out r^2 and place remaining terms on a common denominator:

$$V = \pi r^2\left(\frac{x^3 - 3x^2h + 3xh^2}{3h^2}\right)$$

Addition of $h^3 - h^3$ to the numerator of the fraction does not change its value. The 3 in the denominator can be moved outside:

$$V = \frac{1}{3}\pi r^2\left(\frac{h^3 + x^3 - 3x^2h + 3xh^2 - h^3}{h^2}\right)$$

Grouping the last four terms of the numerator together, the equation can be simplified to:

$$V = \frac{1}{3}\pi r^2\left[\frac{h^3 + (x - h)^3}{h^2}\right]$$

As h is always greater or equal to x, the equation can be written in its final form:

$$V = \frac{1}{3}\pi r^2\left[h - \frac{(h - x)^3}{h^2}\right]$$

where

r = radius of base of flask in cm,
h = height of flask in cm,
x = height of liquid in cm, and
V = volume of liquid in mL.

Proof: At $x = h$; $V = \frac{1}{3}\pi r^2 h$

which is the formula for the volume of a cone.

Summary

Simple general equations have been derived to calculate volumes in two of the most commonly found shapes of laboratory glassware. Calculation of partial volumes in other shapes of vessels are also possible using principles of integral calculus.

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References

[1] Texas Controlled Substances Act; Health and Safety Code; Section 481.160 (1989).
[2] Courtney, M., "Procedure for Volume Estimation in Clandestine Laboratory Reaction Vessels.

Part II, "*Southwest Association of Forensic Scientists Journal*," Volume 12, No. 1, April 1990, pp. 24–30.
[*3*] Ely, R. A., "A Spreadsheet Program for the Determination of Volumes of One and Two Phase Liquids in Round Bottom Flasks," *Microgram* Volume XXIV, No. 7, July 1991, pp. 180–185.

Address requests for reprints or additional information to
John N. Lomonte, M.S.
Southwestern Institute of Forensic Sciences
P.O. Box 35728
Dallas, TX 75235-0728